METHOD OF THE CONTROLLED CURRENT REGULATION – CALCULATION OF PARTIAL CONSTANTS FOR CYLINDRICAL ELECTRODES

METODA KONTROLOVANÉ REGULACE PROUDU – VÝPOČET DÍLČÍCH KONSTANT PRO CYLINDRICKÉ ELEKTRODY

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Abstract

This paper makes possible an exact calculation of the partial constants for the cylindrical electrodes. This calculation supposes the current and potential electrodes and acting of the current electrode on the potential one. It depends on such geometrical factors as the spacing of tool and its diameter, the length of the potential electrode and its diameter and, too, the length of the current electrode and its diameter are. This paper analyses different variances being usual and less usual, but permissible too. It is studied array for Laterolog mainly for 3-electrode one. However, what is very important is that the derived partial cylindrical constants can be too used not only for Laterolog, but for Induced polarization and SP-potentials when you work with the focused electric field. It depends only on algorithm of calculation for the constant of electrode tool. That is different for every of the mentioned well-logging methods.

Abstrakt

Tato práce umožňuje přesný výpočet dílčích konstant pro válcové elektrody. Výpočet předpokládá, že existuje proudová a potenciální elektroda, a že proudová elektroda působí na elektrodu potenciální. Závisí to na takových geometrických faktorech, jako jsou délka sondy, průměr sondy, délka potenciálové elektrody a průměr této elektrody, délka proudové elektrody a také průměr této elektrody. Práce analyzuje různé varianty, které jsou obvyklé i méně obvyklé, ale povolené. Studuje elektrodové uspořádání pro Laterolog, zejména pro 3elektrodový. Avšak co je velmi důležité, je to, že odvozené dílčí konstanty válcového typu mohou být také použity nejen u metody Laterologu, ale i u metody vynucené polarizace a metody spontánní polarizace, když pracujete s usměrněným elektrickým polem. Závisí to na algoritmu výpočtu konstanty uspořádání elektrod hlubinné sondy. Ten je rozdílný u každé ze zmíněných karotážních metod.

Keywords

potential electrode, current electrode, slenderness ratio, the constant of the electrode tool, partial constants, well-logging

Klíčová slova

potenciální elektroda, proudová elektroda, štíhlostní poměr, konstanta hlubinné sondy s elektrodami, dílčí konstanty, karotáž

1 Introduction

Calculation of partial constants of the cylindrical electrodes is made for macro-electrical registration of well-logging data there where the focused electric field is created. It holds especially for various types of Laterolog as the 9-electrode Laterolog, the 7-electrode Laterolog and the 3-electrode Laterolog are. However, the focused electric field can be also used for registration of induced polarization and for static and selective SP-potentials. This presents that calculation of the partial constants for cylindrical electrodes is common for all above well-logging methods, further depends on algorithm of counting of constant for the method and electrode array yet.

2 Derivation of formulas for continuous cylindrical electrodes

The above partial constant depends on geometry of tool. This is given in the first instance by spacing of tool, i.e., by distance being between centres of both electrodes. Next there are dimensions of the current and potential electrodes there. Here belongs the length of the potential electrode and its diameter and the length of the current electrode and its diameter, too. Because of easier derivation it will be supposed the common diameter both for both electrodes and the tool, how it is in fig.1 there.

The idea of calculation is obvious from scheme depicted in fig.1. The first step is you select an arbitrary point on the surface of the cylindrical current electrode. There in that point is the centre of Cartesian variables (x, y, z). You have to calculate contribution of the single current point on all surface of the cylindrical potential electrode.

The second step follows in implement of the next Cartesian variables (k, h, w). The centre of new system is laid into the centre of cylinder presenting the potential electrode. Then you must add simultaneously all contributions of all current points forming the surface of the current electrode.

The starting formula for next calculations is this:

$$dU_0 = \frac{1}{4\pi} \times \left(\frac{R \times I}{r} \right) \times \frac{dS}{S_n}, \tag{1}$$

$$S_n = \pi \times a \times n. \tag{2}$$

It needs to be expressed U_0:

$$U_0 = \frac{1}{4\pi} \times \frac{R \times I}{S_n} \iint_{S_n} \frac{dS}{\sqrt{x^2 + y^2 + z^2}} = \frac{1}{2\pi} \times \left(\frac{R \times I}{a} \right) \times \frac{1}{8} \times \left(\frac{\pi}{2} \right)^{-1} \times \left(\frac{n}{2} \right)^{-1} \iint_{S_n} \frac{dS}{\sqrt{x^2 + y^2 + z^2}}. \tag{3}$$

For a plane element of the cylindrical plane dS it is valid:

$$dS = \frac{a}{2} \times d\varphi \times dz. \tag{4}$$

The z-axis of variables (x, y, z) can be in arbitrary line of the cylindrical surface; however, it is advantageous to select that line in such way to hold for the circle defined generally like

$$(x-k)^2+(y-h)^2=\left(\frac{a}{2}\right)^2, \text{ and} \qquad (5)$$

$$k^2+h^2=\left(\frac{a}{2}\right)^2, \qquad (6)$$

that for k = 0 it holds that h = a/2. Equation (6) is about abscissas k and h, not about the dashed system of coordinates remarked as (k, h, w). It presents condition that the centre of the coordinate system (x, y, z) will be on the surface of the current electrode A.

In such case equation (5) changes its form on this:

$$x^2+\left(y-\frac{a}{2}\right)^2=\left(\frac{a}{2}\right)^2. \qquad (7)$$

Thanks to equation (7) you will get:

$$x^2+y^2=2\times\left(\frac{a}{2}\right)\times y \text{ and} \qquad (8)$$

$$U_0=\frac{1}{4\pi}\times\frac{R\times I}{S_n}\iint_{S_n}\frac{dS}{\sqrt{x^2+y^2+z^2}}=\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{8}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{n}{2}\right)^{-1}\times \iint_{S_n}\frac{dS}{\sqrt{2\times\left(\frac{a}{2}\right)\times y+z^2}}. \qquad (9)$$

Now, you can use the cylindrical variables for x and y:

$$x=\frac{a}{2}\times\cos\varphi,\ y=\frac{a}{2}\times\sin\varphi \qquad (10)$$

If you use formulas (4) and (10) and apply them in equation (9), you will attain this expression:

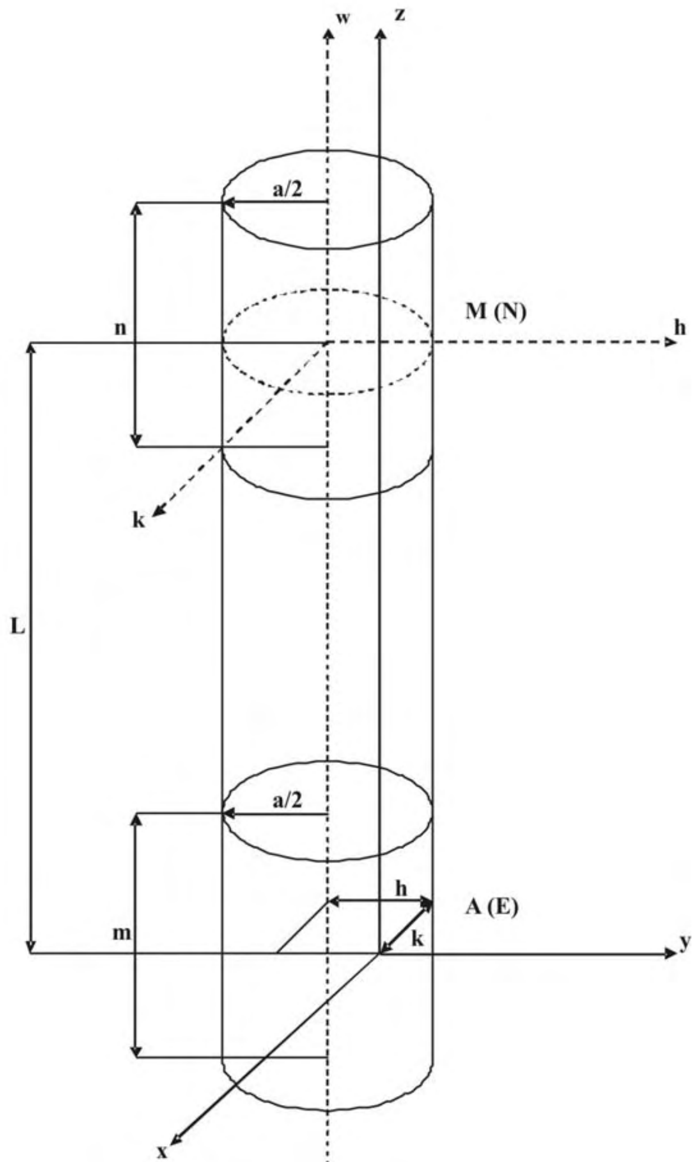

***Fig.1** The cylindrical electrode system presented with two 3-dimensional Cartesian systems of variables (x, y, z) and (k, h, w)*

$$U_0 = \frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{8}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{n}{2}\right)^{-1}\times\left(\frac{a}{2}\right)\int_0^{\pi}\int_{(z-n/2)}^{(z+n/2)}\frac{dz\,d\varphi}{\sqrt{2\times\left(\frac{a}{2}\right)^2\times\sin\varphi+z^2}}. \tag{11}$$

Equation (11) must be adjusted like this:

$$U_0 = \frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{8}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{n}{2}\right)^{-1}\times\int_0^{\pi}\int_{(z-n/2)}^{(z+n/2)}\frac{dz\,d\varphi}{\sqrt{2\times\sin\varphi+\left(\frac{2z}{a}\right)^2}}. \tag{12}$$

If you make substitution that p = (2z/a), you will obtain this expression:

$$U_0 = \frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{8}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{n}{2}\right)^{-1}\times\left(\frac{a}{2}\right)\int_0^{\pi}\int_{(2z/a-n/a)}^{(2z/a+n/a)}\frac{dp\,d\varphi}{\sqrt{2\times\sin\varphi+p^2}}. \tag{13}$$

It holds that:

$$\int_0^{\pi}\frac{dp\,d\varphi}{\sqrt{2\times\sin\varphi+p^2}} = 2\int_0^{\pi/2}\frac{dp\,d\varphi}{\sqrt{2\times\sin\varphi+p^2}}. \tag{14}$$

If you substitute from equation (14) into (13), you will expect this expression:

$$U_0 = \frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{8}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{n}{2}\right)^{-1}\times\left(\frac{a}{2}\right)\times 2\int_0^{\pi/2}\int_{(2z/a-n/a)}^{(2z/a+n/a)}\frac{dp\,d\varphi}{\sqrt{2\times\sin\varphi+p^2}}. \tag{15}$$

This double integral had been solved out with the help of theory of the complex variable. The result of solution is following:

$$\begin{aligned}U_0 = {} & \frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{2}\times\left(\frac{n}{2}\right)^{-1}\times\left(\frac{a}{2}\right)\times\left\{\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2z}{a}+\frac{n}{a}\right)\right]-\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2z}{a}-\frac{n}{a}\right)\right]\right\}\\ & +\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{4}\times\left(\frac{n}{2}\right)^{-1}\times\left(\frac{a}{2}\right)\times\left\{\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2z}{a}+\frac{n}{a}\right)\right]-\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2z}{a}-\frac{n}{a}\right)\right]\right\}.\end{aligned} \tag{16}$$

This formula presents contribution of arbitrary current point being on surface of the current cylindrical electrode. Now, you need to lay the centre of Cartesian system (k, h, w) into the centre of the cylinder, where the axis w is parallel to the axis z, presenting the potential electrode and to integrate along on all surface of the current electrode.

The change of voltage remarked as dU generated with potential U_0 is defined like this:

$$dU = U_0 \times \frac{dS}{S_m}$$, on condition that for U_0 holds z = r where the variable r presents radius – vector. It is allowed if holds that $z \gg \frac{a}{2}$. (17)

$$S_m = \pi \times a \times m, \tag{18}$$

$$\frac{1}{S_m} = \frac{1}{\pi \times a \times m} = \frac{1}{8} \times \left(\frac{\pi}{2}\right)^{-1} \times \left(\frac{m}{2}\right)^{-1} \times \left(\frac{a}{2}\right)^{-1}, \text{ and} \tag{19}$$

$$dS = \left(\frac{a}{2}\right) \times d\varphi \times dw.$$

It was said that in the formula (16) there is now replaced the variable z with radius-vector r. If I express potential U, I will receive that:

$$U = \frac{1}{8} \times \left(\frac{\pi}{2}\right)^{-1} \times \left(\frac{m}{2}\right)^{-1} \iint_{S_m} U_0 \, dw \, d\varphi. \tag{20}$$

You are now in the cylindrical system of coordinates marked as (k, h, w). The cylindrical plane on along there is integrated is directed by circle expressed like that:

$$k^2 + h^2 = \left(\frac{a}{2}\right)^2. \tag{21}$$

With the help of formula (21) you are able to determine that radius-vector r defined as follows:

$$r = \sqrt{k^2 + h^2 + w^2} = \sqrt{\left(\frac{a}{2}\right)^2 + w^2} = \left(\frac{a}{2}\right) \times \sqrt{1 + \left(\frac{2w}{a}\right)^2}. \tag{22}$$

Due to this equation it follows out that there is possible to write:

$$\left(\frac{2r}{a}\right) = \sqrt{1 + \left(\frac{2w}{a}\right)^2}. \tag{23}$$

Equation (16) was said the variable z to have been replaced by the new variable r. Instead of ratio (2z/a) you have (2r/a). If you express that new ratio with the help of formula (23), you will attain the new expression:

$$
\begin{aligned}
U = {} & \frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{16}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{m}{2}\right)^{-1}\times\left(\frac{n}{2}\right)^{-1}\times\left(\frac{a}{2}\right)\int_{0}^{2\pi}\int_{(L-m/2)}^{(L+m/2)}\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+\left(\frac{2w}{a}\right)^{2}}+\left(\frac{n}{a}\right)\right]dw\,d\varphi \\
& -\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{16}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{m}{2}\right)^{-1}\times\left(\frac{n}{2}\right)^{-1}\times\left(\frac{a}{2}\right)\int_{0}^{2\pi}\int_{(L-m/2)}^{(L+m/2)}\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+\left(\frac{2w}{a}\right)^{2}}-\left(\frac{n}{a}\right)\right]dw\,d\varphi \\
& +\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{32}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{m}{2}\right)^{-1}\times\left(\frac{n}{2}\right)^{-1}\times\left(\frac{a}{2}\right)\int_{0}^{2\pi}\int_{(L-m/2)}^{(L+m/2)}\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+\left(\frac{2w}{a}\right)^{2}}+\left(\frac{n}{a}\right)\right]dw\,d\varphi \\
& -\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{32}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{m}{2}\right)^{-1}\times\left(\frac{n}{2}\right)^{-1}\times\left(\frac{a}{2}\right)\int_{0}^{2\pi}\int_{(L-m/2)}^{(L+m/2)}\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+\left(\frac{2w}{a}\right)^{2}}-\left(\frac{n}{a}\right)\right]dw\,d\varphi\,.
\end{aligned}
\tag{24}
$$

If you do integration after φ and use substitution t = (2w/a), you will get this expression:

$$
\begin{aligned}
U = {} & +\left(\frac{R\times I}{a}\right)\times\frac{1}{16}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{m}{a}\right)^{-1}\times\left(\frac{n}{a}\right)^{-1}\int_{(2L/a-m/a)}^{(2L/a+m/a)}\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+t^{2}}+\left(\frac{n}{a}\right)\right]dt \\
& -\left(\frac{R\times I}{a}\right)\times\frac{1}{16}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{m}{a}\right)^{-1}\times\left(\frac{n}{a}\right)^{-1}\int_{(2L/a-m/a)}^{(2L/a+m/a)}\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+t^{2}}-\left(\frac{n}{a}\right)\right]dt \\
& +\left(\frac{R\times I}{a}\right)\times\frac{1}{32}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{m}{a}\right)^{-1}\times\left(\frac{n}{a}\right)^{-1}\int_{(2L/a-m/a)}^{(2L/a+m/a)}\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+t^{2}}+\left(\frac{n}{a}\right)\right]dt \\
& -\left(\frac{R\times I}{a}\right)\times\frac{1}{32}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{m}{a}\right)^{-1}\times\left(\frac{n}{a}\right)^{-1}\int_{(2L/a-m/a)}^{(2L/a+m/a)}\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+t^{2}}-\left(\frac{n}{a}\right)\right]dt\,.
\end{aligned}
$$

This integral needs to implement next substitutions as q = t – (2L/a – m/a), p = $(2m/a)^{-1}$×q and p = 1/s.

$$U = -\left(\frac{R\times I}{a}\right)\times\frac{1}{8}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{n}{a}\right)^{-1}\int_{\infty}^{1} s^{-2}\times\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+\left\{2\left(\frac{m}{a}\right)\times\frac{1}{s}+\frac{2L}{a}-\frac{m}{a}\right\}^{2}}+\left(\frac{n}{a}\right)\right]ds$$

$$+\left(\frac{R\times I}{a}\right)\times\frac{1}{8}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{n}{a}\right)^{-1}\int_{\infty}^{1} s^{-2}\times\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+\left\{2\left(\frac{m}{a}\right)\times\frac{1}{s}+\frac{2L}{a}-\frac{m}{a}\right\}^{2}}-\left(\frac{n}{a}\right)\right]ds$$

$$-\left(\frac{R\times I}{a}\right)\times\frac{1}{16}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{n}{a}\right)^{-1}\int_{\infty}^{1} s^{-2}\times\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+\left\{2\left(\frac{m}{a}\right)\times\frac{1}{s}+\frac{2L}{a}-\frac{m}{a}\right\}^{2}}+\left(\frac{n}{a}\right)\right]ds$$

$$+\left(\frac{R\times I}{a}\right)\times\frac{1}{16}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{n}{a}\right)^{-1}\int_{\infty}^{1} s^{-2}\times\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+\left\{2\left(\frac{m}{a}\right)\times\frac{1}{s}+\frac{2L}{a}-\frac{m}{a}\right\}^{2}}-\left(\frac{n}{a}\right)\right]ds\,.$$

These integrals must be adjusted in the following form:

$$U = +\left(\frac{R\times I}{a}\right)\times\frac{1}{8}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{n}{a}\right)^{-1}\int_{1}^{\infty} s^{-2}\times\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+\left\{2\left(\frac{m}{a}\right)\times\frac{1}{s}+\frac{2L}{a}-\frac{m}{a}\right\}^{2}}+\left(\frac{n}{a}\right)\right]ds$$

$$-\left(\frac{R\times I}{a}\right)\times\frac{1}{8}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{n}{a}\right)^{-1}\int_{1}^{\infty} s^{-2}\times\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+\left\{2\left(\frac{m}{a}\right)\times\frac{1}{s}+\frac{2L}{a}-\frac{m}{a}\right\}^{2}}-\left(\frac{n}{a}\right)\right]ds$$

$$+\left(\frac{R\times I}{a}\right)\times\frac{1}{16}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{n}{a}\right)^{-1}\int_{1}^{\infty} s^{-2}\times\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+\left\{2\left(\frac{m}{a}\right)\times\frac{1}{s}+\frac{2L}{a}-\frac{m}{a}\right\}^{2}}+\left(\frac{n}{a}\right)\right]ds$$

$$-\left(\frac{R\times I}{a}\right)\times\frac{1}{16}\times\left(\frac{\pi}{2}\right)^{-1}\times\left(\frac{n}{a}\right)^{-1}\int_{1}^{\infty} s^{-2}\times\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{1+\left\{2\left(\frac{m}{a}\right)\times\frac{1}{s}+\frac{2L}{a}-\frac{m}{a}\right\}^{2}}-\left(\frac{n}{a}\right)\right]ds\,.$$

All integrals are solved with the help of the complex variable. As final result of such solution here is following expression:

$$U = +\left(\frac{R\times I}{a}\right)\times\frac{1}{8}\times\left(\frac{n}{a}\right)^{-1}\times\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left\{\frac{2L}{a}+\frac{m}{a}\right\}^{2}+1}+\left(\frac{n}{a}\right)\right]$$

$$-\left(\frac{R\times I}{a}\right)\times\frac{1}{8}\times\left(\frac{n}{a}\right)^{-1}\times\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left\{\frac{2L}{a}+\frac{m}{a}\right\}^{2}+1}-\left(\frac{n}{a}\right)\right]$$

$$+\left(\frac{R\times I}{a}\right)\times\frac{1}{16}\times\left(\frac{n}{a}\right)^{-1}\times\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left\{\frac{2L}{a}+\frac{m}{a}\right\}^{2}+1}+\left(\frac{n}{a}\right)\right] \tag{25}$$

$$-\left(\frac{R\times I}{a}\right)\times\frac{1}{16}\times\left(\frac{n}{a}\right)^{-1}\times\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left\{\frac{2L}{a}+\frac{m}{a}\right\}^{2}+1}-\left(\frac{n}{a}\right)\right].$$

Now, you are able to express the constant of cylindrical electrodes:

$$\left(\frac{k}{a}\right)=\frac{1}{F_1+F_2}, \tag{26}$$

$$F_1=\frac{1}{8}\times\left(\frac{n}{a}\right)^{-1}\times\left\{\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left\{\frac{2L}{a}+\frac{m}{a}\right\}^{2}+1}+\left(\frac{n}{a}\right)\right]-\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left\{\frac{2L}{a}+\frac{m}{a}\right\}^{2}+1}-\left(\frac{n}{a}\right)\right]\right\}, \tag{27}$$

$$F_2=\frac{1}{16}\times\left(\frac{n}{a}\right)^{-1}\times\left\{\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left\{\frac{2L}{a}+\frac{m}{a}\right\}^{2}+1}+\left(\frac{n}{a}\right)\right]-\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left\{\frac{2L}{a}+\frac{m}{a}\right\}^{2}+1}-\left(\frac{n}{a}\right)\right]\right\}, \tag{28}$$

where (L/a) = slenderness ratio of the tool,
(m/a) = slenderness ratio of the current electrode, and
(n/a) = slenderness ratio of the potential electrode.

In fig.1 there was supposed an identical diameter of both electrodes and the tool, as well. Reality used to be other. Cylindrical electrodes can have the same diameter, but, the tool has usually its diameter a bit bigger. Generally speaking the diameters can be various. Nevertheless, in spite of that you can use the derived formulas – only tiny adjusted.

$$\left(\frac{k}{a_L}\right)=\frac{1}{F_1+F_2}, \tag{29}$$

$$F_1=\frac{1}{8}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{2L}{a_L}+\frac{m}{a_m}\right)^2+1}+\frac{n}{a_n}\right]-\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{2L}{a_L}+\frac{m}{a_m}\right)^2+1}-\frac{n}{a_n}\right]\right\}, \tag{30}$$

$$F_2=\frac{1}{16}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{2L}{a_L}+\frac{m}{a_m}\right)^2+1}+\frac{n}{a_n}\right]-\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{2L}{a_L}+\frac{m}{a_m}\right)^2+1}-\frac{n}{a_n}\right]\right\} \tag{31}$$

where L = distance being between both centres of the current and potential electrodes [m],
m = length of the current electrode [m],
n = length of the potential electrode [m],
a_L = diameter of the tool body [m],
a_m = diameter of the current electrode [m], and
a_n = diameter of the potential electrode [m].

3 Formulas for segmented electrodes

The potential electrodes can be segmented into several parts. The metallic segments are isolated between themselves by material having high resistance. This is made for elimination of the electrode potentials creating on the surface of electrodes. However, under insulator there are all segments conductively connected and they form the single electrode. This is done because of automatic discharge of the electrode potentials when the above segments are continuously discharged one to other. Therefore it is important not to create a leakage of insulator between segments, because each of the electrode segments and water present the separate electric cell.

The current electrodes can be segmented if they are used like the potential electrodes, as well. This is possible for the 3-electrode Laterolog, Induced polarization or SP-potentials, for example. The constant of the electrode tool in such case is calculated due to the next formula going out of i-th segment of electrode.

$$\left(\frac{k}{a_L}\right)=\frac{1}{n}\times\sum_{i=1}^{n}\frac{1}{F_{1i}+F_{2i}}=\frac{1}{n}\times\sum_{i=1}^{n}\left(\frac{k}{a_L}\right)_i, \tag{32}$$

where n = the number of segments.

Formula (32) presents separate calculation for each of segments and their average. More about the segmented electrodes is in RYŠAVÝ, F. (2006). Formula (32) holds too for segmented electrode when is A≡M, possibly, E≡N.

4 Analysis of formulas having been derived for partial cases

This analysis investigates formulas remarked as (29), (30) and (31). These equations reflect relations being between the size of electrodes given by length and diameter and the tool spacing, see fig.2. Here they are:

$$\left(\frac{k}{a_L}\right)=\frac{1}{F_1+F_2},$$

$$F_1=\frac{1}{8}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{2L}{a_L}+\frac{m}{a_m}\right)^2+1}+\frac{n}{a_n}\right]-\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{2L}{a_L}+\frac{m}{a_m}\right)^2+1}-\frac{n}{a_n}\right]\right\},$$

$$F_2=\frac{1}{16}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{2L}{a_L}+\frac{m}{a_m}\right)^2+1}+\frac{n}{a_n}\right]-\mathrm{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{2L}{a_L}+\frac{m}{a_m}\right)^2+1}-\frac{n}{a_n}\right]\right\}$$

There must be accepted the fundamental inequality:

$$\left(\frac{2L}{a_L}\right)>\left(\frac{m}{a_m}\right)+\left(\frac{n}{a_n}\right). \qquad (33)$$

It is clear that for $a_L = a_m = a_n = a$, it holds that L > (m/2) + (n/2). That inequality (33) has two characteristic stages. The first is when it holds:

$$\left(\frac{2L}{a_L}\right)>>\left(\frac{m}{a_m}\right)+\left(\frac{n}{a_n}\right).$$

This presents that stage when the distance being between both centres of electrodes will be much longer than dimensions of both electrodes are. The second is possible to present as relation:

$$\left(\frac{2L}{a_L}\right)\to\left(\frac{m}{a_m}\right)+\left(\frac{n}{a_n}\right).$$

Such relation means that the distance being between both centres of electrodes is almost comparable to dimensions of both electrodes. However, you have to distinguish two cases again; for $(m/a_m) << (n/a_n)$ and for $(m/a_m) >> (n/a_n)$. All three main stages are depicted in fig.2.

a. Stage remarked as $(2L/a_L) >> (m/a_m) + (n/a_n)$

You have to use simpler conditions:

$$\left(\frac{2L}{a_L}+\frac{m}{a_m}\right)\approx\frac{2L}{a_L},\quad\sqrt{\left(\frac{2L}{a_L}\right)^2+1}\approx\frac{2L}{a_L}.$$

Then formulas characterizing terms F_1 and F_2 are following:

$$F_1=\frac{1}{8}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2L}{a_L}\right)+\frac{n}{a_n}\right]-\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2L}{a_L}\right)-\frac{n}{a_n}\right]\right\},\text{and} \quad (34)$$

$$F_2=\frac{1}{16}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\text{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2L}{a_L}\right)+\frac{n}{a_n}\right]-\text{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2L}{a_L}\right)-\frac{n}{a_n}\right]\right\}. \quad (35)$$

In this stage there is neglected an influence of dimensions of the current electrode. This is case of Pseudolaterolog and Classical Laterolog.

Partial stage remarked as $(2L/a_L)\rightarrow(m/a_m)+(n/a_n)$

You have to implement the only simpler condition:

$$\left(\frac{2L}{a_L}\right)\approx\left(\frac{m}{a_m}\right)+\left(\frac{n}{a_n}\right).$$

The formulas for F_1 and F_2 will be following:

$$F_1=\frac{1}{8}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{2m}{a_m}+\frac{n}{a_n}\right)^2+1}+\frac{n}{a_n}\right]-\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{2m}{a_m}+\frac{n}{a_n}\right)^2+1}-\frac{n}{a_n}\right]\right\}, \quad (36)$$

$$F_2=\frac{1}{16}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\text{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{2m}{a_m}+\frac{n}{a_n}\right)^2+1}+\frac{n}{a_n}\right]-\text{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{2m}{a_m}+\frac{n}{a_n}\right)^2+1}-\frac{n}{a_n}\right]\right\}. \quad (37)$$

This stage neglects an influence of the distance between both centres of the current and potential electrodes. Significance of dimensions of both electrodes is more than obvious. This case is somewhere in between the 3-electrode Laterolog and 7-electrode Laterolog (Classical Laterolog).

b. Stage remarked as $(2L/a_L)\rightarrow(m/a_m)+(n/a_n)$ *and* $(m/a_m)<<(n/a_n)$

Into equations (36) and (37) we implement simpler condition that:

$$\left(\frac{2m}{a_m}+\frac{n}{a_n}\right)\approx\frac{n}{a_n}.$$

Then terms F_1 and F_2 are following:

$$F_1=\frac{1}{8}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{n}{a_n}\right)^2+1}+\frac{n}{a_n}\right]-\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{n}{a_n}\right)^2+1}-\frac{n}{a_n}\right]\right\}, \tag{38}$$

$$F_2=\frac{1}{16}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\text{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{n}{a_n}\right)^2+1}+\frac{n}{a_n}\right]-\text{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{n}{a_n}\right)^2+1}-\frac{n}{a_n}\right]\right\}. \tag{39}$$

Besides that fact that there is neglected an influence of tool dimensions, there is neglected, too, the effect of the current electrode. What is important is that dimensions of the potential electrode prevail over dimensions of the current electrode. It is characteristic for the 3-electrode Laterolog when the guard electrodes are much longer than the length of the central electrode; however what is more important is that formulas (38) and (39) are characteristic too for the case when both potential and current electrodes are identical. It is again about 3-electrode Laterolog; more is in chapter 5.

c. Stage remarked as $(2L/a_L) \to (m/a_m) + (n/a_n)$ *and* $(m/a_m) >> (n/a_n)$

We use again equations (36) and (37). Here are conditions making simpler terms F_1 and F_2:

$$\left(\frac{2m}{a_m}+\frac{n}{a_n}\right)\approx\frac{2m}{a_m},\quad \sqrt{\left(\frac{2m}{a_m}\right)^2+1}\approx\frac{2m}{a_m}.$$

The terms F_1 and F_2 will gain this form:

$$F_1=\frac{1}{8}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2m}{a_m}\right)+\frac{n}{a_n}\right]-\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2m}{a_m}\right)-\frac{n}{a_n}\right]\right\}, \text{ and} \tag{40}$$

$$F_2=\frac{1}{16}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\text{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2m}{a_m}\right)+\frac{n}{a_n}\right]-\text{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2m}{a_m}\right)-\frac{n}{a_n}\right]\right\}. \tag{41}$$

In this case there is neglected an influence of the tool dimensions only. The effect of dimensions of the potential and current electrodes remains. Such array is unusual, but possible. I am sure you have noted that all partial cases remarked as a., b. and c. in fig.2 confirm that the effect of the potential electrode is impossible to neglect.

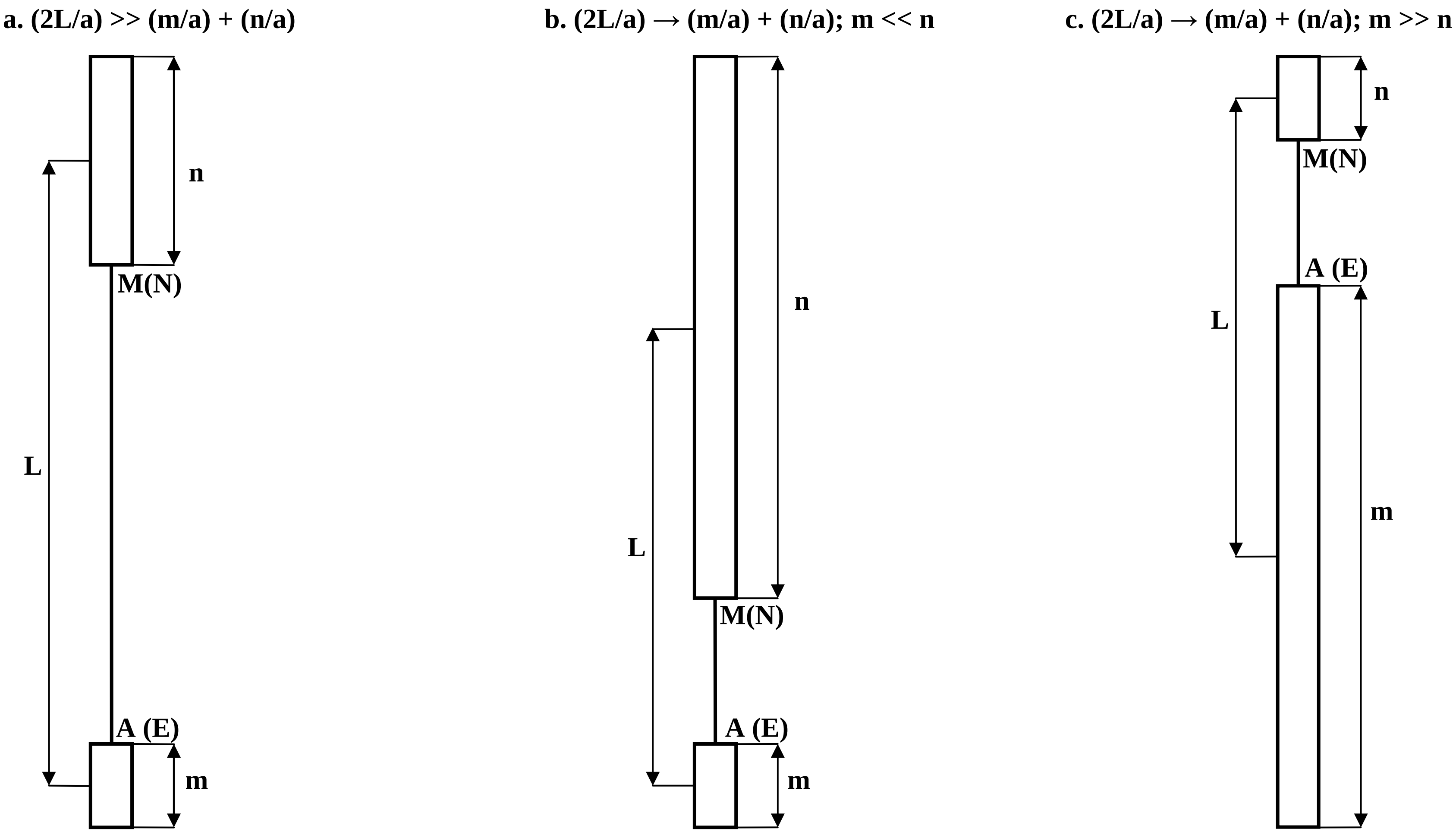

Fig.2 The partial stages having various slenderness ratios of the current and potential electrodes and tool

The next big case is when both potential and current electrodes have common identity; that means that L = 0 what present that A ≡ M and E ≡ N; however, this is theme of next chapter.

5 Derivation of formulas when the potential electrode is simultaneously the current one

Such case is not anything special. It is visible for the 3-electrode Laterolog; however, the method of induced polarization can have the common electrode too. You have to implement into formulas (29), (30) and (31) conditions that:

$\left(\frac{2L}{a_L}\right)=0$, and $\left(\frac{m}{a_m}\right)=\left(\frac{n}{a_n}\right)$.

The second condition is a bit special. This condition is filled when it holds that m = μ×n and a_m = μ× a_n when μ = const. For μ = 1 there are both electrodes fully dimensionally identical. Generally, they can have various dimensions; nevertheless, their slenderness ratios will be identical. After substitution into the above formulas you will get:

$$\left(\frac{k}{a_n}\right)=\frac{1}{F_1+F_2}, \qquad (42)$$

$$F_1=\frac{1}{8}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{n}{a_n}\right)^2+1}+\frac{n}{a_n}\right]-\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{n}{a_n}\right)^2+1}-\frac{n}{a_n}\right]\right\}, \qquad (43)$$

$$F_2=\frac{1}{16}\times\left(\frac{n}{a_n}\right)^{-1}\times\left\{\operatorname{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{n}{a_n}\right)^2+1}+\frac{n}{a_n}\right]-\operatorname{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\sqrt{\left(\frac{n}{a_n}\right)^2+1}-\frac{n}{a_n}\right]\right\}. \qquad (44)$$

As these formulas remarked as (43) and (44) are identical to ones remarked as (38) and (39) it is evident that this case is equivalent to that remarked as stage b.

6 Derivation of the limit relation for the point electrodes

All the before derived formulas incline to the basic relation; this relation presents formula of the asymptote. The formula is for the point electrodes.

You have to use formula (29) being adjusted like that:

$$\left(\frac{k}{a_L}\right)=\frac{1}{F_1+F_2}=\frac{2\pi}{2\pi\times(F_1+F_2)}. \qquad (45)$$

Further, we use formulas (34) and (35). Because in these formulas condition of the point electrodes is filled for the current electrode yet, you add the next condition for the potential electrode; $(n/a_n)\rightarrow 0$. Then you have to calculate this limit:

$$F_1=\frac{1}{8}\times\lim_{(n/a_n)\to 0}\frac{\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2L}{a_L}+\frac{n}{a_n}\right)\right]-\ln\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2L}{a_L}-\frac{n}{a_n}\right)\right]}{\left(\frac{n}{a_n}\right)}=\frac{1}{8}\times\lim_{(n/a_n)\to 0}\frac{\ln\left[\frac{2L}{a_L}+\frac{n}{a_n}\right]-\ln\left[\frac{2L}{a_L}-\frac{n}{a_n}\right]}{\left(\frac{n}{a_n}\right)}=$$

$$=\frac{1}{8}\times\lim_{(n/a_n)\to 0}\frac{\left[\frac{2L}{a_L}+\frac{n}{a_n}\right]^{-1}+\left[\frac{2L}{a_L}-\frac{n}{a_n}\right]^{-1}}{1}=\frac{1}{8}\times 2\times\left[\frac{2L}{a_L}\right]^{-1}=\frac{1}{4}\times\left[\frac{2L}{a_L}\right]^{-1}.$$

$$F_2=\frac{1}{16}\times\lim_{(n/a_n)\to 0}\frac{\text{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2L}{a_L}+\frac{n}{a_n}\right)\right]-\text{Argsinh}\left[\left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{2L}{a_L}-\frac{n}{a_n}\right)\right]}{\left(\frac{n}{a_n}\right)}=$$

$$=\frac{1}{16}\times\lim_{(n/a_n)\to 0}\frac{\left(\frac{\sqrt{2}}{2}\right)\times\left[1+\frac{1}{2}\times\left(\frac{2L}{a_L}+\frac{n}{a_n}\right)^2\right]^{-\frac{1}{2}}+\left(\frac{\sqrt{2}}{2}\right)\times\left[1+\frac{1}{2}\times\left(\frac{2L}{a_L}-\frac{n}{a_n}\right)^2\right]^{-\frac{1}{2}}}{1}=$$

$$=\frac{1}{16}\times\frac{\sqrt{2}}{2}\times\frac{2}{\sqrt{1+\frac{1}{2}\times\left(\frac{2L}{a_L}\right)^2}}=\frac{1}{16}\times\sqrt{\frac{2}{1+\frac{1}{2}\times\left(\frac{2L}{a_L}\right)^2}}.$$

Now, we have to introduce the condition that $(2L/a_L)\to\infty$. In such case the line of the electrode tool presents geometrical body–it is infinitely slender cylinder.

$$\lim_{(2L/a_L)\to\infty} 2\pi\times\left(F_1+F_2\right)=\lim_{(2L/a_L)\to\infty} 2\pi\times\left\{\frac{1}{4}\times\left[\frac{2L}{a_L}\right]^{-1}+\frac{1}{16}\times\sqrt{\frac{2}{1+\frac{1}{2}\times\left(\frac{2L}{a_L}\right)^2}}\right\}=$$

$$=\lim_{(2L/a_L)\to\infty}\frac{\pi}{2}\times\left\{\left[\frac{2L}{a_L}\right]^{-1}+\frac{1}{4}\times\sqrt{4\times\left(\frac{2L}{a_L}\right)^{-2}}\right\}=\lim_{(2L/a_L)\to\infty}\frac{\pi}{2}\times\left\{\left[\frac{2L}{a_L}\right]^{-1}\times\left(1+\frac{1}{2}\right)\right\}=\lim_{(2L/a_L)\to\infty}\frac{3}{4}\pi\times\left[\frac{2L}{a_L}\right]^{-1}=$$

$$=\left[\frac{2L}{a_L}\right]^{-1} \text{ for } L>>a_L\,.$$

After substitution into equation (45) you receive:

$$\left(\frac{k}{a_L}\right)=\frac{2\pi}{\left[\frac{2L}{a_L}\right]^{-1}}=2\pi\times\left[\frac{2L}{a_L}\right]=4\pi\times\left[\frac{L}{a_L}\right] \text{ for } L>>a_L. \tag{46}$$

It is obvious that it holds that:

$$k=4\pi\times L\,. \tag{47}$$

The limit record in formula (46) must fill condition of the point electrodes: the cylindrical body of tool tends to line, because the line in three-dimensional space is nothing less than an infinitely slender cylinder. It confirms validity of the before derived formulas and the basic formula remarked as (47). Mathematical processes used for derivation of needed formulas draw upon experience of the following authors: ŠKRÁŠEK, J., TICHÝ, Z. (1983), ŠKRÁŠEK, J., TICHÝ, Z. (1986).

7 Discussion to partial constants

Very important is to realize that dimensions of electrode array are different not only for variances of Laterolog, but too for methods SP-potentials and Induced polarization. Even if producer has usually same the length of all electrodes for 9-electrode Laterolog and for 7-electrode Laterolog, the distance between the current and potential electrodes is within one electrode array variable. And for 3-electrode Laterolog there is the main that the central electrode is much more less than both guard electrodes. However, this paper offers an exact count of partial constants within one electrode array. It is all new procedure, other than that used by I.MARUŠIAK being described in

MARUŠIAK, I. (1968) and MARUŠIAK, I., TĚŽKÝ, A., JONÁŠOVÁ, V (1969). Here you work with partial constants and their geometry. The constants are all exactly quantified and this tends to exact calibration.

It considers about constants remarked as k_{AM}, k_{AN}, k_{EM} and k_{EN}; these are for 7-electrode Laterolog and 3-electrode Laterolog. The 9 -electrode Laterolog, so called Pseudo-Laterolog, has six partial constants: k_{AM}, k_{AN}, k_{EM}, k_{EN}, k_{BM} and k_{BN}. And for methods as the induced polarization and SP-potentials are, is situation very similar to. Through the derived formulas for counting you can determine all partial constants; because you need them for determination of the constant of the electrode tool remarked as K and for the coefficient of focusing η. Algorithms of counting of both mentioned factors are various after variances of method and the sort of geophysical method. Generally said, it holds that $K = f(k_{AM}, k_{AN}, k_{EM}, k_{EN}, k_{BM}, k_{BN})$ and $\eta = f(k_{AM}, k_{AN}, k_{EM}, k_{EN}, k_{BM}, k_{BN})$. More to this you can find in the paper RYŠAVÝ, F. (2013).

8 Conclusions

Thanks to analysis of formulas before derived here are the following conclusions:

- The partial constant of cylindrical electrodes is exactly calculable. It depends on geometry defined by slenderness ratios of tool and the current and potential electrodes. This way needs not to have a tank with water having the known resistivity.
- In partial cases there is possible to neglect an influence of tool and the current electrode. However, the influence of the potential electrode always exists.
- If the potential electrode and the current electrode present the common one it is also possible to compute the constant of tool; there hold conditions $L = 0$, $m = n$ and $a_m = a_n$.
- On account of elimination of the electrode potentials the potential electrode can be segmented into several parts. If the potential and current electrodes present the only common one it is possible, as well.
- The partial constants are useable not only for Laterolog, but too for Induced polarization and for SP-potentials. The condition is a focused electric field.
- Thanks to the derived formulas you can determine partial constants of the concrete electrode array. The mentioned constants remarked as k_{AM}, k_{AN}, k_{EM}, k_{EN}, k_{BM} and k_{BN} are important for counting of the constant of the electrode tool, K, and for the coefficient of focusing η.

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